DETERMINATION OF THE MOISTURE CONTENT OF CAPILLARY-POROUS MATERIALS FROM MICROWAVE ABSORPTION

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The dependence of the damping of microwaves in moist capillary-porous materials on their moisture content is investigated theoretically. Expressions are obtained for the absolute and relative moisture content.

Experimental investigations show [1-5] that there exists a stable functional relation between the microwave energy losses in moist materials and their moisture content. However, the theoretical foundation for microwave moisture determination methods remain at the level of empirical formulas.

In an analytic examination of the relation between the damping of microwaves in moist material and the moisture content of the latter, most specialists working in this field use the well-known Maxwell, Lorenz–Lorentz, Lichtenecker, and Odelevskii equations and their various modifications, which make it possible to calculate the bulk content of each phase of a multicomponent mixture from its average dielectric constant [2, 3, 6].

Such an approach to the solution of this problem is connected with the traditional representation of capacitive moisture-content measurement, for which the formal treatment of the equations indicated above is valid. An analysis of [6, 7, 10, 13] shows that the solution of this problem has a physical meaning only in the case of the electrodynamic approach to the description of the processes involving interaction of microwaves with moist capillary-porous material.

As is well known [9], the interaction of an electromagnetic field with matter is described by the equations of macroscopic electrodynamics. When microwaves pass through moist capillary-porous materials, electric and magnetic dipoles, quadrupoles, and multipoles are excited in each structure phase. The energy drawn in this case by a particle from the incident wave goes to heating (absorption of the microwave energy) and secondary radiation (scattering of the microwave energy). The losses depend on the dimensions and shape of the particles of the corresponding structure phases, on the wavelength, on the complex dielectric constant, on the temperature, and on the placement and number of particles. The calculation is based on the theory of scattering and absorption of electromagnetic waves by particles of simplest form, such as spheres or cylinders, whose dimensions are smaller than the wavelength [11-13]. To simplify the calculations we assume that the particles are randomly distributed, that the scattered radiation is incoherent, and that the concentration and statistical distribution of the particles are constant over the entire path traversed by the radio waves.

The moisture inclusions and the components of the dry skeleton of the capillary-porous material will be characterized by the effective scattering and absorption cross sections

$$\sigma_s = \frac{W_s}{W_0}, \quad \sigma_a = \frac{W_a}{W_0}. \tag{1}$$

For a three-phase material we find from (1) that the amount of energy scattered and absorbed on a path element dx is given by

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TABLE 1. Values of ε', ε", and tan δ for Certain Dry Materials

Material	λ, cm	ε'	ε"	tg δ	Material	λ, cm	ε'	ε"	tg δ
Red brick	10	3,4	0,085	0,025	Sand	3	3,2	0,06	0,02
Gypsum	10	2,65	0,0135	0,005	Sand	3	3,0	0,04	0,013
Cement	10	2,1	0,0735	0,035	Ice	3	3,17	0,0022	0,0007
Lumber	10	2,5	0,175	0,07	Snow	3	1,26	0,0005	0,0004
Lumber	3	2,6	0,39	0,15	Paper	3	2,7	0,135	0,05
Cement	3	4,0	0,08	0,02	Grease	3	2,5	0,125	0,05
Asbestos	3	2,0	0,12	0,06	Nylon	3	2,8	0,029	0,011
Sand	9	2,0	0,04	0,02					

$$dW_s' = -N'\sigma_s' W_0 dx, \quad dW_a' = -N'\sigma_a' W_0 dx,$$
$$dW_s'' = -N''\sigma_s'' W_0 dx, \quad dW_a'' = -N''\sigma_a'' W_0 dx, \tag{2}$$
$$dW_s''' = -N'''\sigma_s''' W_0 dx, \quad dW_a''' = -N'''\sigma_a''' W_0 dx.$$

Integrating (2), we obtain the microwave energy losses, in decibels, to scattering and absorption after traversing a path x:

$$K_s' = 4.343 \cdot 10^2 \cdot x\Sigma N'\sigma_s', \quad K_a' = 4.343 \cdot 10^2 \cdot x\Sigma N'\sigma_a',$$
$$K_s'' = 4.343 \cdot 10^2 \cdot x\Sigma N''\sigma_s'', \quad K_a'' = 4.343 \cdot 10^2 \cdot x\Sigma N''\sigma_a'', \tag{3}$$
$$K_s''' = 4.343 \cdot 10^2 \cdot x\Sigma N'''\sigma_s''', \quad K_a''' = 4.343 \cdot 10^2 \cdot x\Sigma N'''\sigma_a'''.$$

To calculate the attenuation it is necessary to take into account the coefficient of reflection from the interface between the material and the air

$$|r|^2 = \frac{4r^2 \sin^2 \Phi}{(1-r)^2 + 4r^2 \sin \Phi}. \tag{4}$$

The total loss following passage of microwaves through the moist material is obtained by summing (3) and (4)

$$K_\Sigma = 10 \lg |r|^2 + 4.343 \cdot 10^2 \cdot x (\Sigma N'\sigma_s' + \Sigma N'\sigma_a' + \Sigma N''\sigma_s'' + \Sigma N''\sigma_a'' + \Sigma N'''\sigma_s''' + \Sigma N'''\sigma_a'''). \tag{5}$$

To calculate the total effective cross section for scattering by inhomogeneities of different shapes, it is necessary to solve the diffraction problem. Such a solution entails considerable difficulties, and in most cases one uses approximate methods based on the use of the Kirchhoff–Heugens principle. Analogous problems were solved in radar to calculate the effective cross sections of scattering by targets of different types [14]. For bodies with very simple shape, rigorous calculations were based on solving Maxwell's equations at specified boundary conditions [15]. In the case of dielectrics, such as the structural components of the capillary-porous materials, the solution becomes more complicated. For a sphere of radius a_0, having a complex dielectric constant ε^*, the effective scattering cross section is determined in terms of a slowly converging series in Legendre polynomials, and does not lend itself readily to calculations in the general case. In the scalar case, the problem of scattering by the dielectric sphere has much in common with the quantum-mechanical problem of scattering by a potential barrier at high energy [16]. It should be noted that the main effects are determined in this case not by diffraction but by relfection and refraction [17], as is the case in our problem.

Assuming that the inhomogeneities in the structure of the material and the moisture inclusions are spherical in form, and recognizing that their dimensions are smaller than the wavelength of the electromagnetic radiation, we write formulas for the calculation of the effective scattering and absorption cross sections [10, 13-16],

$$\sigma_{ss} = \frac{\pi a_0^2}{\rho^2} |(a_1^2 - b_1^2) + 5b_2^2|,$$
$$\sigma_{as} = \frac{\lambda^2}{2\pi} \rho^3 (c_1 + c_2\rho^2 + c_3\rho^3). \tag{6}$$

Assuming $\rho = 2\pi a_0/\lambda < 1$, we solve (6). After the calculations we obtain

$$\sigma_{ss} = \frac{64\pi^5 a_0^6}{\lambda^4} \cdot \left| \frac{\varepsilon^* - 1}{\varepsilon^* + 2} \right|^2, \tag{7}$$

TABLE 2. Values of ε', $\tan\delta$, and c_1^* for Water at Different Temperatures and Wavelengths

T, °C	λ=1,2 cm			λ=3 cm			λ=10 cm		
	ε'	tgδ	c_1^*	ε'	tgδ	c_1^*	ε'	tgδ	c_1^*
1,5	15	0,425	0,862	38	1,03	2,983	80,5	0,31	1,386
5,0	17,5	0,395	0,881	41	0,95	2,885	80,2	0,275	1,229
15	25	0,330	0,860	49	0,70	2,376	78,8	0,205	0,912
25	34	0,265	0,795	55	0,54	1,971	76,7	0,157	0,691
35	41	0,215	0,704	58	0,44	1,662	74,0	0,127	0,551
45	46	0,275	1,391	59	0,40	1,530	70,7	0,106	0,425
55	49	0,245	0,869	60	0,36	1,391	67,5	0,089	0,370
65	50,5	0,125	0,454	59	0,32	1,231	64,0	0,076	0,310
75	51,5	0,105	0,381	57	0,28	1,062	60,5	0,066	0,261
85				54	0,26	0,963	56,5	0,055	0,209

$$\sigma_{as} = \frac{4\pi^2 a_0^3}{\lambda}\left[\frac{6\varepsilon''}{(\varepsilon'+2)^2+(\varepsilon'')^2}\right]. \tag{8}$$

Substituting in (5) the effective cross sections for a sphere, we obtain

$$\begin{aligned} K_\Sigma = 10\lg|r|^2 + 4.343\cdot 10^2\cdot x \Big\{\Sigma N' \frac{64\pi^5 a_1^6}{\lambda^4} \\ \times\left|\frac{\varepsilon_1^*-1}{\varepsilon_1^*+2}\right|^2 + \Sigma N' \frac{4\pi^2 a_1^3}{\lambda}\left[\frac{6\varepsilon_1''}{(\varepsilon_1'+2)^2+(\varepsilon_1'')^2}\right] + \Sigma N'' \frac{64\pi^5 a_2^6}{\lambda^4} \\ \times\left|\frac{\varepsilon_2^*-1}{\varepsilon_2^*+2}\right|^2 + \Sigma N'' \frac{4\pi^2 a_2^3}{\lambda}\left[\frac{6\varepsilon_2''}{(\varepsilon_2'+1)^2+(\varepsilon_2'')^2}\right] + \Sigma N''' \frac{64\pi^5 a_3^6}{\lambda^4} \\ \times\left|\frac{\varepsilon_3^*-1}{\varepsilon_3^*+2}\right|^2 + \Sigma N''' \frac{4\pi^2 a_3^3}{\lambda}\left[\frac{6\varepsilon_3''}{(\varepsilon_3'+1)^2+(\varepsilon_3'')^2}\right]. \end{aligned} \tag{9}$$

Recognizing that in a microwave band the energy loss in the dry material (Table 1) is smaller by more than two orders of magnitude than in water (Table 2), we write (9) in the form

$$K_\Sigma' = 4.343\cdot 10^2 xQ\,\frac{3\pi^2}{\lambda}\left[\frac{16\pi^3 a_3^3}{\lambda^3}\left|\frac{\varepsilon_3^*-1}{\varepsilon_3^*+2}\right|^2 + \frac{6\varepsilon_3''}{(\varepsilon_3'+2)^2+(\varepsilon_3'')^2}\right]. \tag{10}$$

Since $a_3 \ll \lambda$, the first terms in the brackets of (10) can be neglected. We then obtain

$$K_\Sigma'' = 4.343\cdot 10^2 xQ\,\frac{3\pi^2}{\lambda}\left[\frac{6\varepsilon_3''}{(\varepsilon_3'+2)^2+(\varepsilon_3'')^2}\right]. \tag{11}$$

At constant λ, ε_3'', ε_3', and x, the relative attenuation of the microwaves in the moist material is proportional to the cube of the linear dimension of the included moisture, i.e., to the volume of the water in the pores of the dry matter.

Equation (11) expresses the fact that radio-wave absorption is determined by the amount of water in the material. To establish an analytic connection between the microwave energy loss and the moisture contents of capillary-porous materials, let us consider a layer of moist material of thickness x. Taking (11) into account, we can conclude that the loss in moist material of thickness x is equivalent to the microwave energy loss in a layer of water of thickness x_1 (accurate to 0.1%)

$$K_{x_1} = 4.343 x_1 \frac{2\pi}{\lambda}\left(\frac{4\varepsilon_3'^2\varepsilon_3'' - \varepsilon_3''^2}{8\varepsilon_3'^2\sqrt{\varepsilon_3'}}\right). \tag{12}$$

The weights of the water, dry and moist material per unit volume are respectively

$$P_w = x_1\gamma_w S, \tag{13}$$

$$P_{dm} = (x - x_1)\gamma_{dm} S, \tag{14}$$

$$P_{mm} = x\gamma_{mm} S. \tag{15}$$

TABLE 3. Calculated and Experimental Results of the Measurement of the Moisture Content of Different Materials by the Microwave Absorption Method (λ = 3 cm, T = 25°C)

Material	x, cm	K_Σ, db	Relative moisture content, %: calculation	Relative moisture content, %: from the experimental data
Asbestos cement	0,6	10	13	15 (R. A. Berentsveig)
The same	0,6	15	22	20 "
Wheat	32,5	15	12,4	13 (H. B. Taylor)
The same	32,5	20	15,6	15,7 "
" "	32,5	30	22	20 "
Nylon	55	25	4,6	5 "
The same	55	36	6	6,2 "
Coal (pulverised)	7,5	7,5	4,6	5 "
The same	7,5	12	7,3	8 "
" "	7,5	20	13	15 "
Sand	25	60	7,2	7 (V. K. Benzar')
The same	25	50	6,1	6 "
" "	10	60	19	20 "
" "	10	50	16	16,2 "
" "	10	20	6	6,3 "
Lumber	20	29	10	12 "
The same	20	58	20	21 "
" "	20	80	30	28,5 "
Brick (red)	11	16,7	5	8 "
The same	11	31	10	12 "
" "	11	58	20	19,5 "
Peat (milled)	12	10	20	16,5 "
The same	12	5	10	9,6 "
" "	12	20	40	37 "

From (13), (14), and (15) we obtain expressions for the absolute and relative moisture content

$$U_a = \frac{P_w}{P_{dm}} = \frac{x_1\gamma_w}{(x - x_1)\gamma_{dm}}, \tag{16}$$

$$U_{rel} = \frac{P_w}{P_{mm}} = \frac{x_1\gamma_w}{x_1\gamma_w + (x - x_1)\gamma_{dm}}. \tag{17}$$

Substituting in (16) and (17) the values of x_1 from (12), we obtain

$$U_a = \frac{K'_\Sigma}{x\cdot 4.343\frac{2\pi}{\lambda}c_1^* - K^1_\Sigma}\,\frac{\gamma_w}{\gamma_{dm}}, \tag{18}$$

$$U_{rel} = \frac{K'_\Sigma}{K'_\Sigma + \frac{\gamma_{dm}}{\gamma_w}\left(x\cdot 4.343\,\frac{2\pi}{\lambda}\,c_1^* - K'_\Sigma\right)}. \tag{19}$$

The experimental data on the determination of the moisture content of capillary-porous materials, obtained by different authors and calculated from formulas (18) and (19), are summarized in Table 3. The agreement can be regarded as satisfactory if allowance is made for the methodological errors, for the different procedures, and for the influence of the reflection from the interface between the air and material, which were not taken into account in the calculation.

NOTATION

σ_s, σ_a are the effective scattering and absorption cross sections;
W_0, W_a, W_s are the energies of the incident, absorbed, and scattered waves;
N', N'', N''' are the amount of included dry matter, air, and moisture;
N' + N'' + N''' = 1;
$r = (\sqrt{\varepsilon} - 1)(\sqrt{\varepsilon} + 1)$ is the coefficient of reflection from the front surface of the material;
x is the thickness of the material;
λ is the wavelength;
K', K'', K''' are the microwave energy losses in the corresponding phases;
K_Σ is the total loss in decibels;
K_Σ = K' + K'' + K''';
σ_{ss}, σ_{as} are the effective cross sections for a sphere;

ε_1^*, ε_2^*, ε_3^* are the complex dielectric constants of the corresponding structure phases;

$\varepsilon^* = \varepsilon' + \varepsilon''$;

c_1, c_2, c_3 are the functions of the dielectric constant;

$Q = (4/3)\pi a^3 N$ is the water content per unit volume of the material;

x_1 is the thickness of water layer;

$$c_1^* = \frac{(4\,\varepsilon_3'^2\,\varepsilon_3' - \varepsilon_3''^2)}{8\varepsilon_3'^2 \sqrt{\varepsilon_3'}};$$

S is the area of sample;

γ_w, γ_{dm}, γ_{mm} are the weights of water, dry matter, and moist matter per unit volume;

$$\Phi = \frac{2\pi x}{\lambda}\sqrt{\varepsilon^* - \sin^2\theta}.$$

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